

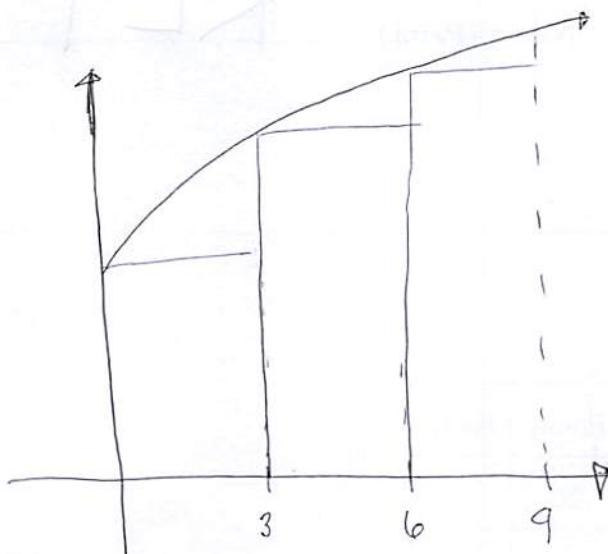
Math 241
Spring 2018
Exam 3 - Practice
4/9/18
Time Limit: 50 Minutes

Name (Print):

KEY

Problem	Points	Score
1	20	
2	20	
3	50	
4	15	
5	15	
6	20	
7	20	
Total:	160	

1. (20 points) a) In the space below, draw a reasonable sketch of $f(x) = \sqrt{x} + 1$ and draw 3 equal width rectangles associated to the lower (Riemann) sum approximation on the interval $[0, 9]$.



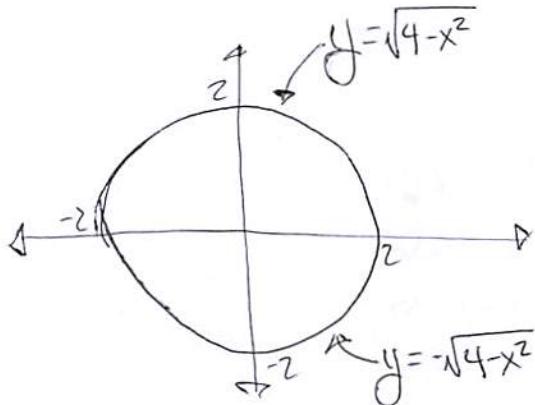
- b) Express the area above in sigma notation.

$$\sum_{k=0}^2 3 \left(3(k-1)^{1/2} + 1 \right)$$

- c) Is this an ~~overestimate~~ or an ~~underestimate~~?

(By definition of the lower sum!)

2. (a) (10 points) Set up a definite integral which gives the area of a circle with radius 2.



$$\int_{-2}^2 \sqrt{4-x^2} - (-\sqrt{4-x^2}) dx$$

- (b) (10 points) Let $G(x) = \int_1^x 1 - t^2 dt$. Determine the intervals of increase/decrease and concavity for G .

$$G'(x) = 1 - x^2 \quad \text{and} \quad G''(x) = -2x$$

G is increasing on $[-1, 1]$

G is decreasing on $(-\infty, -1] \cup [1, \infty)$

G is C.U. on $(-\infty, 0]$ and C.D. on $[0, \infty)$

$$3. \text{ (a) (10 points)} \int \frac{1}{\sqrt{x}} + \cos(2x+1) + 2 \, dx = \int x^{-1/2} + \cos(2x+1) + 2 \, dx \\ = 2x^{1/2} + \underbrace{\sin(2x+1)}_{2} + 2x + C$$

(b) (10 points) $\int \frac{x+1}{(x^2+2x)^3} \, dx$, $u = x^2+2x$, $du = 2x+2 \, dx$

$\frac{1}{2} \int \frac{1}{u^3} \, du$, $du = 2(x+1) \, dx$, $\frac{1}{2} \, du = x+1 \, dx$

$$= \frac{1}{2} \cdot \frac{u^{-2}}{-2} + C = \frac{-1}{4(x^2+2x)^2} + C$$

(c) (10 points) $\int_0^9 \sqrt{\sin(\pi x)} \cos(\pi x) \, dx$, $u = \sin(\pi x)$, $du = \cos(\pi x) \cdot \pi \, dx$

$\Rightarrow \int_0^0 \sqrt{u} \cdot \frac{1}{\pi} \, du$, $\frac{1}{\pi} \, du = \cos(\pi x) \, dx$

$$= 0$$

(d) (10 points) $\int (x+1)^{50} x \, dx$, $u = x+1$, $du = dx$

$u-1 = x$

$$\int u^{50}(u-1) \, du = \int u^{51} - u^{50} \, du = \frac{u^{52}}{52} - \frac{u^{51}}{51} + C$$

$$= \frac{(x+1)^{52}}{52} - \frac{(x+1)^{51}}{51} + C$$

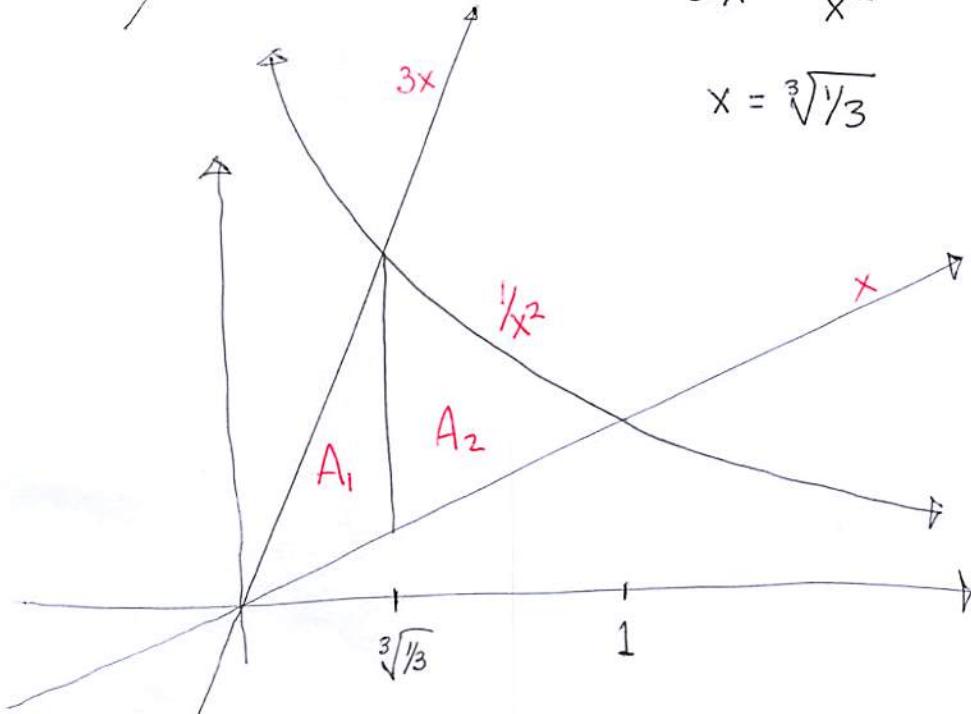
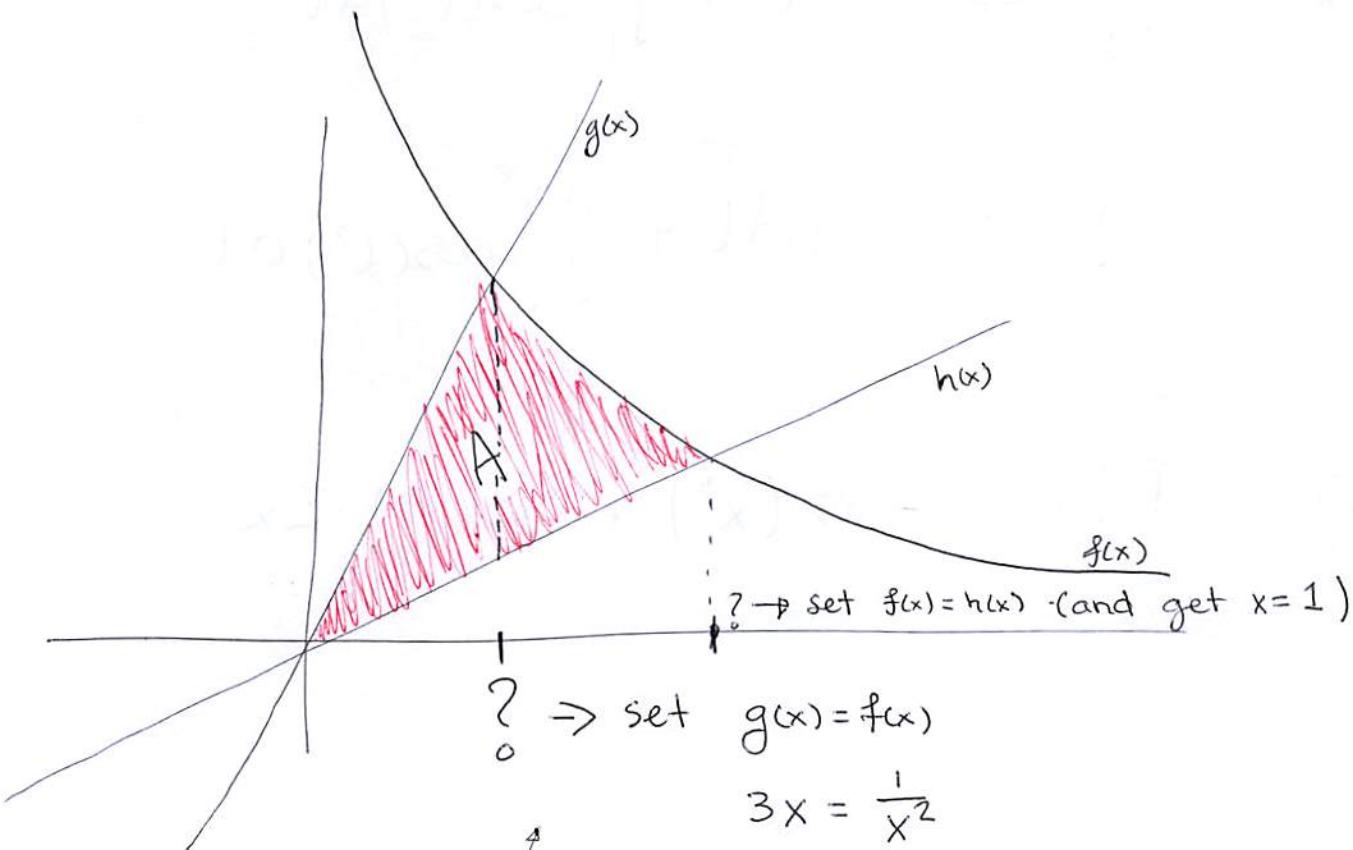
(e) (10 points) $\int \frac{x^2+2x+1}{x^{3/2}} \, dx$

$$= \int \frac{x^2}{x^{3/2}} + \frac{2x}{x^{3/2}} + \frac{1}{x^{3/2}} \, dx$$

$$= \int x^{1/2} + \frac{2}{x^{1/2}} + \frac{1}{x^{3/2}} \, dx$$

$$= \frac{2}{3} x^{3/2} + 4x^{1/2} + \frac{x^{-1/2}}{-1/2} + C$$

4. (15 points) Find the area of the region bounded by $f(x) = \frac{1}{x^2}$, $g(x) = 3x$ and $h(x) = x$.



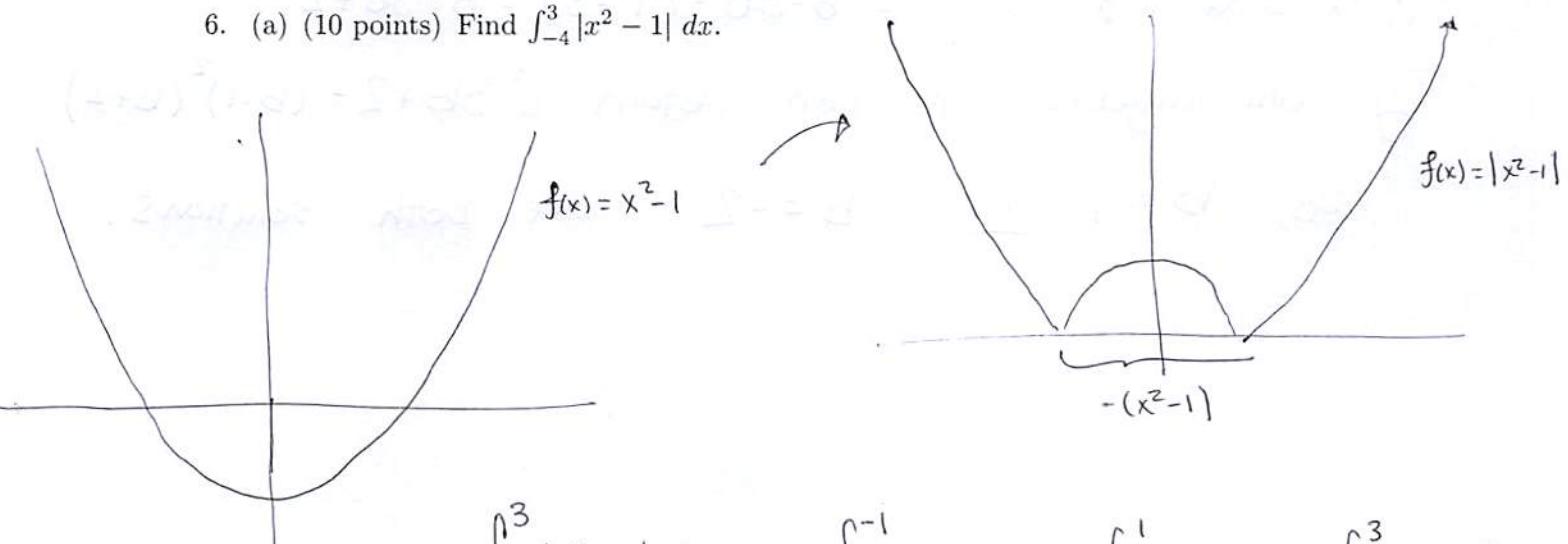
$$A = A_1 + A_2 = \int_0^{\sqrt[3]{1/3}} 3x - x \, dx + \int_{\sqrt[3]{1/3}}^1 \frac{1}{x^2} - x \, dx \quad (\text{I'm feeling lazy})$$

5. (15 points) Let $F(x) = \int_x^{x^2} \cos(t^3) dt$. Determine $F'(x)$.

$$\begin{aligned} F(x) &= \int_x^1 \cos(t^3) dt + \int_1^{x^2} \cos(t^3) dt \\ &= - \int_1^x \cos(t^3) dt + \int_1^{x^2} \cos(t^3) dt \end{aligned}$$

Now, $F'(x) = -\cos(x^3) + \cos(x^6) \cdot 2x$

6. (a) (10 points) Find $\int_{-4}^3 |x^2 - 1| dx$.



$$\int_{-4}^3 |x^2 - 1| dx = \int_{-4}^{-1} x^2 - 1 dx + \int_{-1}^1 1 - x^2 dx + \int_1^3 x^2 + 1 dx$$

(b) (10 points) Find $\int \frac{\sqrt{\sin(x)} + 1 \cos(x)}{\sqrt{\sin(x)}} dx$

Set $u = \sqrt{\sin(x)} + 1$, $du = \frac{\cos(x)}{2\sqrt{\sin(x)}} dx$, $2du = \frac{\cos(x)}{\sqrt{\sin(x)}} dx$

$\star \int \sqrt{u} \cdot 2 du$

$$= 2 \frac{u^{3/2}}{3/2} + C = \frac{4}{3} (\sqrt{\sin(x)} + 1)^{3/2} + C$$

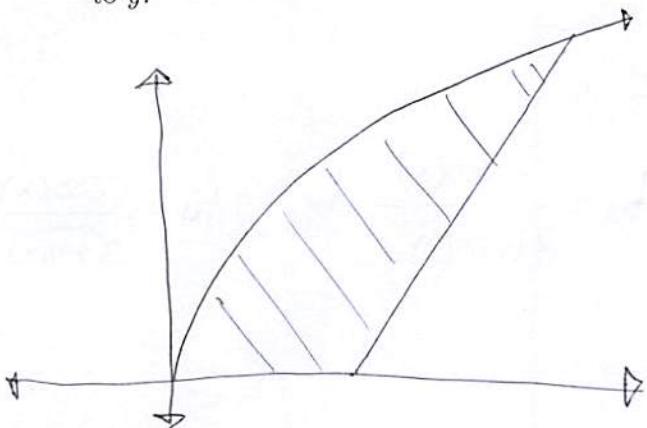
7. (20 points) a) Give all values of b such that $\int_1^b 3x^2 - 3 \, dx = 0$. from here we can see that $b=1$ is a solution.

$$\int_1^b 3x^2 - 3 \, dx = x^3 - 3x \Big|_1^b = b^3 - 3b - (1^3 - 3) = b^3 - 3b + 2$$

By some algebra, one can obtain $b^3 - 3b + 2 = (b-1)^2(b+2)$

so, $b=1$ and $b=-2$ are both solutions.

- b) Set up integrals which give area of the region bounded by the first quadrant and the lines $y = \sqrt{x}$ and $y = x - 2$. One integrating with respect to x and the other integrating with respect to y .



With respect to x :

$$\int_0^2 \sqrt{x} \, dx + \int_2^4 x - 2 \, dx$$

With respect to y :

$$\int_0^2 y + 2 - (y^2) \, dy$$